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EFFECT OF HEAT RELEASE ON THE SPATIAL STABILITY
OF A SUPERSONIC REACTING MIXING LAYER

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ABSTRACT

We have begun a numerical study of the stability of compressible mixing layers in which a diffusion flame is embedded. In this study, we have approximated the mean velocity profile by a hyperbolic tangent profile and taken the limit of infinite activation energy which reduces the diffusion flame to a flame sheet. The addition of combustion in the form of a flame sheet was found to have important, and complex, effects on the flow stability.

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1. Introduction. Quite recently it has been realized that an understanding of the stability characteristics of reacting compressible mixing layers is extremely important in view of the projected use of the scramjet engine for the propulsion of hypersonic aircraft. For example, Drummond and Mukunda (1988) suggest that "... a single supersonic, spatially developing and reacting mixing layer serves as an excellent physical model for the overall flow field." Thus knowledge of the stability characteristics may allow one, in principle, to control the downstream evolution of such flows in the combustor. This is particularly important because of the observed increase in the flow stability at high Mach numbers (Brown and Roshko, 1974; Chinzei, Masuya, Komuro, Murakami, and Kudou, 1986; and Papamoschou and Roshko, 1986). Because of the gain in stability, natural transition may occur at downstream distances which are larger than practical combustor lengths. A number of techniques which may enhance mixing are discussed by Kumar, Bushnell and Hussaini (1987).

Perhaps motivated by this practical problem, there have been a number of theoretical studies of the stability of compressible free shear layers in recent years. These include that of Tam and Hu (1988) who examined the stability of the compressible mixing layer in a channel using the hyperbolic tangent profile. They concluded that the sound waves reflected from the wall have a major effect on the flow stability. Zhuang, Kubota and Dimotakis (1988) also studied the mixing layer with the hyperbolic tangent profile and found decreasing amplification with increasing Mach number. Ragab (1988) carried out a numerical solution of the two dimensional compressible Navier-Stokes equations for the wake/mixing layer behind a splitter plate with the streams on the two sides of the plate having different speeds. In a linear stability analysis of the computed mean flow, he found that increasing the Mach number leads to a strong stabilization of the flow. Ragab and Wu (1988) examined the viscous and inviscid stability of a compressible mixing layer using

both the hyperbolic tangent and Lock profiles. They found that if the Reynolds number was greater than 1000, the disturbances could be calculated very accurately from inviscid theory. In addition, they reported that nonparallel effects are negligible.

Earlier studies of the stability of compressible mixing layers include those of Lessen, et al (1965,1966) and Gropengiesser (1969). The inviscid temporal stability of the compressible mixing layer to two and three dimensional disturbances was studied by Lessen, Foz and Zien for subsonic disturbances (1965) and supersonic disturbances (1966). Lessen et al. assumed that the flow was iso-energetic and, as a consequence, the temperature of the stationary gas was always greater than that of the moving gas. Gropengiesser (1969) reexamined this problem, but without using the iso-energetic assumption. Consequentially, he was able to treat the ratio of the temperatures of the stationary and moving gas as a parameter. His results showed that there was a general decrease in the growth rates of the unstable disturbances (both two and three dimensional) as the Mach number of the flow increased, at least up to a Mach number of 3. He also showed a second unstable mode in a narrow range of Mach numbers, $1.54 < M < 1.73$.

Despite the fact that understanding of the flow field in a *reacting* compressible mixing layer in a scramjet engine is extremely important, there appears to be very few studies on the stability of such flows. Menon, Anderson and Pai (1984) studied the inviscid spatial stability of a compressible wake in which there was a $H_2 - O_2$ reaction. When the reaction was turned on the flow became completely unstable. The phase speed was found to be a monotonically increasing function of frequency. It seems that their results show a complete absence of neutral or stable disturbances.

The above result appears to be in conflict with that of Drummond and Mukunda (1988). They carried out a numerical simulation using the two dimensional, compressible, time dependent Navier-Stokes equations with combustion in a mixing layer. The reaction was the burning of a 10% H_2 , 90% N_2 fuel in air. They found that the nonreacting flow was very stable and that turning on the combustion had little effect. But it should be noted that the authors were not carrying out a stability calculation, per se, and did not excite the flow with disturbances with a fixed frequency. They relied on the "natural" disturbances to perturb the flow. Finally, it should be noted that Drummond and Mukunda found that the Lock profile (Lock, 1951) was an excellent fit to the computed mean velocity profile beginning a short way downstream from the origin of the mixing layer.

We have begun a numerical study of the stability of compressible mixing layers in which a diffusion flame is embedded. The basic steady flow with which we began is that calculated by Jackson and Hussaini (1988). In their study the limit of infinite activation energy was used and the diffusion flame reduced to a flame sheet. The flame sheet model is a standard approximation and has been used in the study of the burning of a fuel particle in an oxidizing atmosphere and of the flame at the mouth of a tube, for example (Buckmaster and Ludford, 1982; and Williams, 1985).

It is well known (see Mack, 1984) that inviscid stability theory is a reliable guide for understanding the stability of compressible shear flows at moderate and large Reynolds numbers ($Re > 10^3$). Thus we considered only the inviscid spatial stability problem. We consider the Mach number range $0 \leq M \leq 10$. Our results suggest that this range of Mach numbers (relative to the moving stream) is sufficient to deduce the asymptotic ($M \rightarrow \infty$) structure of the solutions.

We began our study by taking the mean velocity profile in the mixing layer to be that of the Lock profile. In the course of carrying out the stability calculations for this profile with different sets of values of the basic parameters we noted some quite interesting features of the solutions, particularly at higher Mach numbers. In order to examine these features in more detail, we replaced the Lock profile with a hyperbolic tangent profile. This is a reasonable approximation to the Lock profile and has been used in studying the stability of compressible mixing layers, for example, Tam and Hu (1988), Ragab and Wu (1988), and Zhuang, Kubota and Dimotakis (1988).

In section 2 we give the basic flow profile and the small amplitude disturbance equations. The boundary conditions and numerical method are also discussed in this section. Section 3 contains a presentation of our results and conclusions are given in section 4.

2. Formulation. The nondimensional equations governing the steady two dimensional flow of a compressible, reacting mixing layer which lies between streams of reactants with different speeds and temperatures are given by Jackson and Hussaini (1988). In the limit of infinite activation energy, the thin diffusion flame reduces to a flame sheet. The nondimensional temperature T and mass fractions C_i profiles are given by

$$T = 1 - (1 - \beta_T - \beta)(1-U) + \frac{\gamma-1}{2} M^2 U (1-U), \quad C_1 = 2U - 1, \quad C_2 = 0, \quad (2.1)$$

for $\eta > \eta_f$ in the moving stream, and

$$T = \beta_T + (1 - \beta_T + \beta)U + \frac{\gamma-1}{2} M^2 U (1-U), \quad C_1 = 0, \quad C_2 = 1 - 2U, \quad (2.2)$$

for $\eta < \eta_f$ in the stationary stream. Here, η_f gives the location of the flame sheet where both reactants vanish, β_T is the ratio of the temperature of the stationary stream to that of the moving stream, γ is the ratio of specific heats, and β is the nondimensional heat release parameter. Figure 1 shows the variation of T with η for various values of β . The discontinuity in the slope is due to the flame sheet approximation. Note that if β_T is less than one, the stationary gas is relatively cold compared to the moving stream, and if β_T is greater than one it is relatively hot. As discussed in the introduction, we assume here that

$$U = \frac{1}{2} (1 + \tanh(\eta)), \quad (2.3)$$

which approximates the Lock profile and can be handled analytically.

The flow field is perturbed by introducing wave disturbances in the velocity, pressure, temperature, density and mass fractions on either side of the flame sheet with amplitudes which are functions of η . For example, the pressure perturbation is

$$p = \Pi(\eta) \exp[i(\alpha x - \omega t)], \quad (2.4)$$

with Π the amplitude, α the wavenumber in the downstream (x) direction, and ω the frequency which is taken to be real. Substituting the expression (2.4) for the pressure perturbation and similar expressions for the other flow quantities, it is straightforward to derive a single equation governing Π , given by

$$\Pi'' - \frac{2U'}{U-c} \Pi' - \alpha^2 T [T - M^2(U-c)^2] \Pi = 0, \quad c = \frac{\omega}{\alpha}, \quad (2.5)$$

with c being the complex phase speed and primes indicating differentiation with respect to the similarity variable η . In (2.4), α is complex. The real part of α is the wave number in the x direction, while the imaginary part of α indicates whether the disturbance is amplified, neutral, or damped depending on whether α_i is negative, zero, or positive.

The boundary conditions for Π are obtained by considering the limiting form of equation (2.5) as $\eta \rightarrow \pm \infty$. The solutions to (2.5) are of the form

$$\Pi \rightarrow \exp(\pm \Omega_{\pm} \eta), \quad (2.6)$$

where

$$\Omega_{+}^2 = \alpha^2 [1 - M^2(1-c)^2], \quad \Omega_{-}^2 = \alpha^2 \beta_T [\beta_T - M^2 c^2]. \quad (2.7)$$

Let us define c_{\pm} to be the values of the phase speed for which Ω_{\pm}^2 vanishes. Thus,

$$c_{+} = 1 - 1/M, \quad c_{-} = \sqrt{\beta_T}/M. \quad (2.8)$$

Note that c_{+} is the phase speed of a sonic disturbance in the moving stream and c_{-} is the phase speed of a sonic disturbance in the stationary stream. Across the flame sheet, Π and Π' are continuous.

The nature of the disturbances and the appropriate boundary conditions can now be illustrated by reference to Figure 2, where we plot c_{\pm} versus M . In what follows we assume that $\alpha_r^2 > \alpha_i^2$. These curves divide the $c_r - M$ plane into four regions. If a disturbance exists with a M and c_r in region 1, then Ω_{+}^2 and Ω_{-}^2 are both positive, and the disturbance is subsonic at both boundaries. In region 3, both Ω_{+}^2 and Ω_{-}^2 are negative and hence the disturbance is supersonic at both boundaries. In region 2, Ω_{+}^2 is positive and Ω_{-}^2 is negative, and the disturbance is subsonic at $+\infty$ and supersonic at $-\infty$. Finally, in region 4, Ω_{+}^2 is negative and Ω_{-}^2 is positive so the disturbance is supersonic at $+\infty$ and subsonic at $-\infty$.

One can now see that the appropriate boundary condition for either damped or outgoing waves in the moving and stationary streams are, respectively,

$$\Pi \rightarrow e^{-\Omega_{+} \eta}, \quad \text{if } c > c_{+}, \quad \Pi \rightarrow e^{-i\eta \sqrt{-\Omega_{+}^2}}, \quad \text{if } c < c_{+}, \quad (2.9a)$$

$$\Pi \rightarrow e^{\Omega_{-} \eta}, \quad \text{if } c < c_{-}, \quad \Pi \rightarrow e^{-i\eta \sqrt{-\Omega_{-}^2}}, \quad \text{if } c > c_{-}. \quad (2.9b)$$

To solve the disturbance equation (2.5), we first transform it to a Riccati equation by setting

$$G = \frac{\Pi'}{\alpha T \Pi}. \quad (2.10)$$

Thus, (2.5) becomes

$$G' + \alpha T G^2 - \left[\frac{2U'}{U-c} - \frac{T'}{T} \right] G = \alpha [T - M^2(U-c)^2]. \quad (2.11)$$

The boundary conditions can be found from (2.9) and (2.10), with G continuous across the flame sheet.

The stability problem is thus to solve equation (2.11) for a given real frequency ω and Mach number M , with U and T defined by (2.1)-(2.3). The eigenvalue is the wavenumber α . Because

this equation has a singularity at $U = c$, we shall integrate it along the complex contour $(-6,-1)$ to $(0,0)$ and $(0,0)$ to $(6,-1)$. We iterate on α until the boundary conditions are satisfied.

3. Results. In all of our calculations reported here we have taken $\gamma = 1.4$, $\beta_T = 2$, $0 \leq \beta \leq 5$, and $0 \leq M \leq 10$.

In Figures 3 and 4 we show the phase speeds of the neutral waves and the corresponding maximum growth rates, respectively, for the nonreactive ($\beta = 0$) mixing layer (Jackson and Grosch, 1988). From Figure 3 one can see that there is only a single fast regular neutral mode in region 1 which is subsonic at the boundaries. This mode ceases to exist at the Mach number (M_s) at which its phase speed equals that of a sonic wave. Beyond M_s (region 2) the mode is transformed into a singular neutral mode which is subsonic at one boundary and supersonic at the other. In addition, a slow singular neutral mode appears in region 4. In regions 2 and 4 there are unstable waves with phase speeds between that of the singular neutral mode and that of the sonic neutral mode. Thus, there are two bands of unstable frequencies for Mach numbers greater than M_s . One of these bands is a group of fast and the other a group of slow unstable waves. The phase speeds of both the fast and slow modes have a small range about the average, so that little dispersion of wave packets is expected, with a reduction in the dispersion as the Mach number is increased. From Figure 4 it is apparent that an increase in the Mach number results in a decrease of the growth rate of the fast mode by a factor of three or four up to about M_s , and for higher Mach numbers this growth rate levels off and eventually begins to increase with increasing Mach number. The growth rate of the slow mode is comparable to that of the fast mode for Mach numbers greater than M_s .

The addition of heat release ($\beta > 0$) was found to have important, and complex, effects on the flow stability. In contrast to the nonreacting case, Figure 5 shows the existence of multiple regular neutral modes for $M = 0$. There are two regular modes, fast and slow, and a singular mode adjacent to the slow mode. The fast mode exists for all values of β , but the slow modes only exist for $\beta > 1$. The maximum growth rates for the unstable waves are shown in Figure 6. An increase in β results in a reduction in the growth rate of the fast wave and an increase in the growth rate of the slow wave. For sufficiently large β these curves cross so that the slow mode becomes the most unstable.

The phase speeds of the neutral modes as a function of Mach number for $\beta = 1, 2, 5$ are shown in Figures 7-9. An increase in the value of β causes an increase in the phase speed of the fast neutral mode in region 1. In all cases, the fast regular neutral modes are transformed into singular neutral modes in region 2 at M_s . For higher Mach numbers, there are unstable waves with phase speeds between that of the singular neutral mode and that of the sonic neutral mode. It can be seen that the addition of heat release has a major effect on the slow modes. For $\beta > 1$, there are both regular and singular modes in region 1. Again, there are unstable waves with phase speeds between that of the slow neutral modes in region 1 and between that of the singular neutral mode and the sonic mode in region 4. Even with combustion heating, the maximum growth rates of the unstable waves, shown in Figures 10-12, decrease by a factor of three to four as the Mach number approaches M_s . As in the nonreacting case, for Mach numbers greater than M_s , the growth rates level off and eventually begin to increase with increasing Mach number. The rate at which the

growth rates increase with Mach number is different than in the nonreacting case. Also, increasing the heat release generally increases the growth rate of the slow waves and decreases that of the fast waves.

4. Summary. The addition of combustion in the form of a flame sheet was found to have important, and complex, effects on the flow stability. In contrast to the nonreacting case, we have shown the existence of multiple regular neutral modes for Mach numbers which are less than M_s . For Mach numbers greater than M_s , there are two bands of unstable frequencies (just as in the nonreacting case); one a group of fast waves and one a group of slow waves. The range of phase speeds in each band is small about the average, so even with chemical heating there is little dispersion of wave packets. Even with heating, the maximum growth rates of the unstable waves decrease by a factor of three to four as the Mach number approaches M_s . As in the nonreacting case, for Mach numbers greater than M_s , the growth rates level off and eventually begins to increase with increasing Mach number. The rate at which the growth rates increase with Mach number is different than in the nonreacting case. The overall effect of increasing β is to first stabilize the flow and then to destabilize it.

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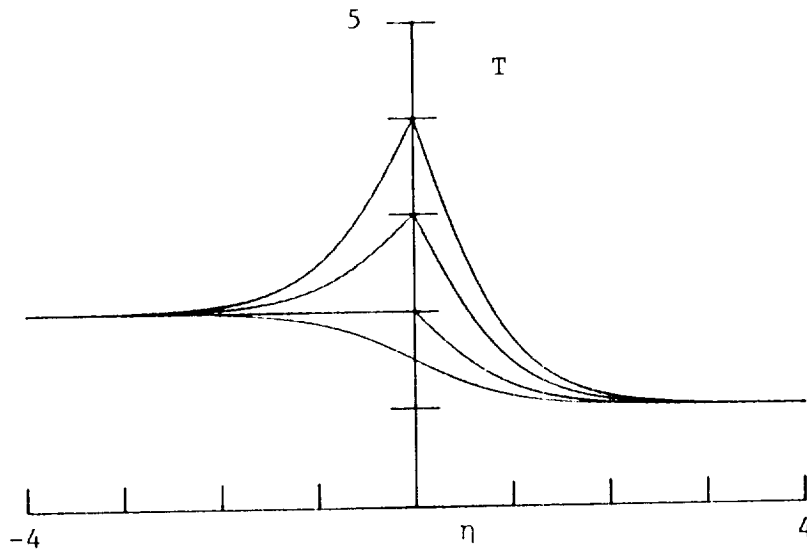


Figure 1. Plot of temperature T versus η for $\beta = 0, 1, 3, 5$ and $M = 0$.

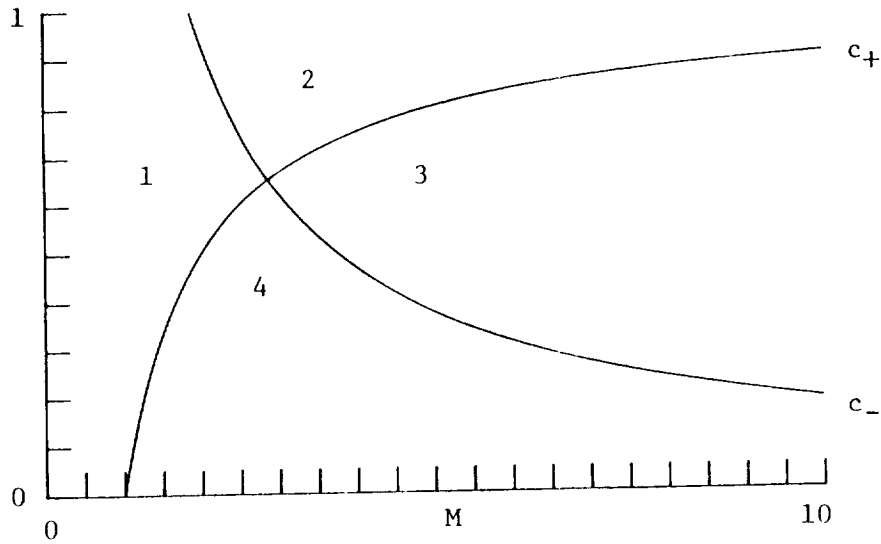


Figure 2. Plot of the sonic speeds c_{\pm} versus Mach number.

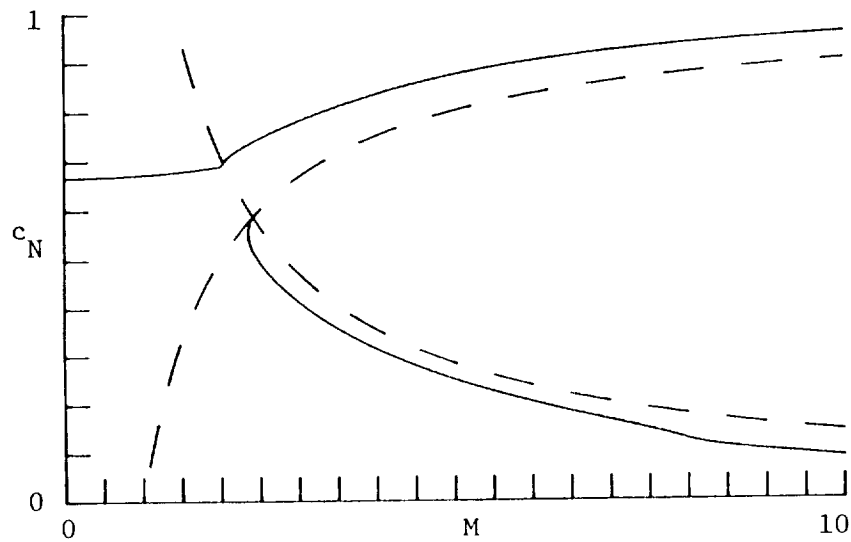


Figure 3. Plot of neutral phase speeds (solid) and sonic speeds (dashed) versus Mach number for $\beta = 0$.

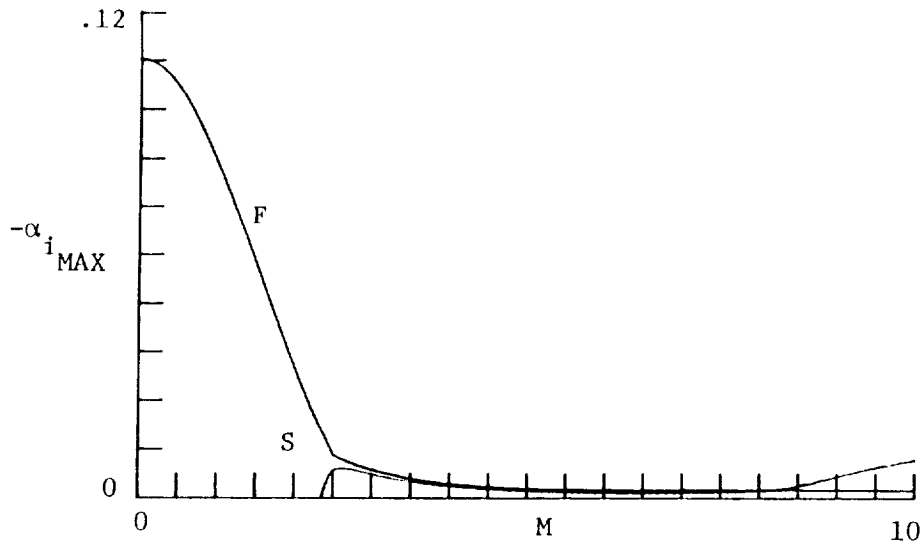


Figure 4. Plot of maximum growth rate versus Mach number for $\beta = 0$.

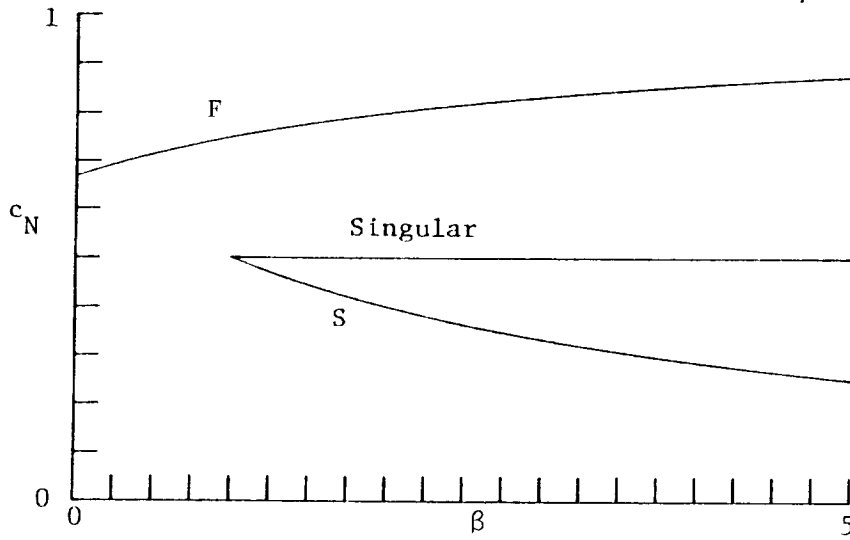


Figure 5. Plot of neutral phase speeds versus β for $M = 0$.

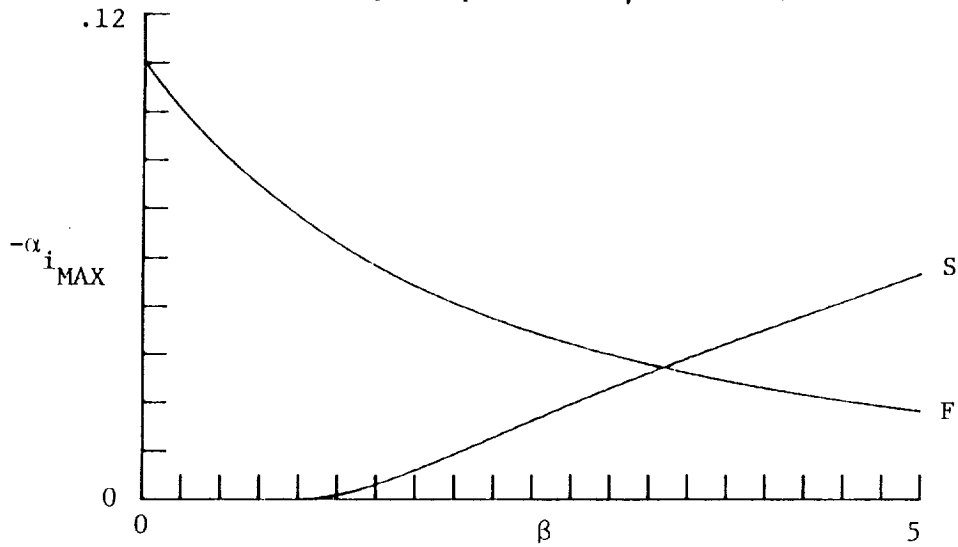


Figure 6. Plot of maximum growth rates versus β for $M = 0$.

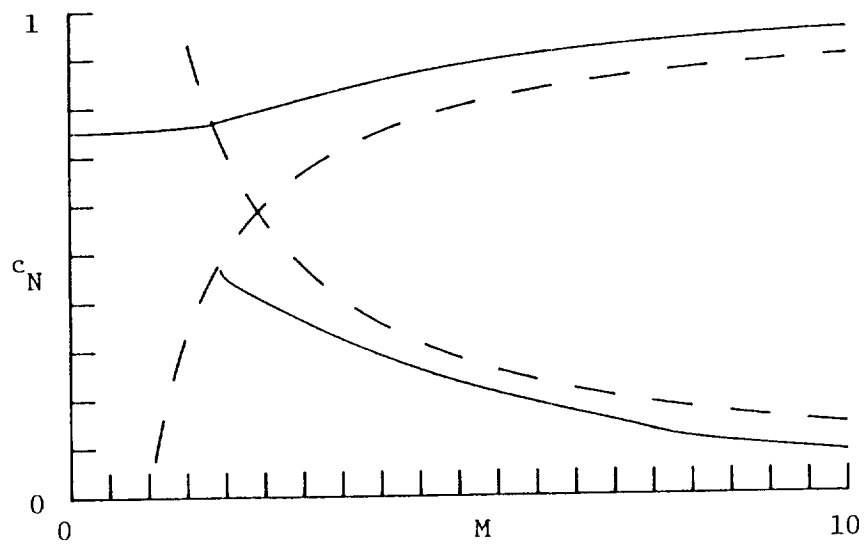


Figure 7. Plot of neutral phase speeds versus Mach number for $\beta = 1$.

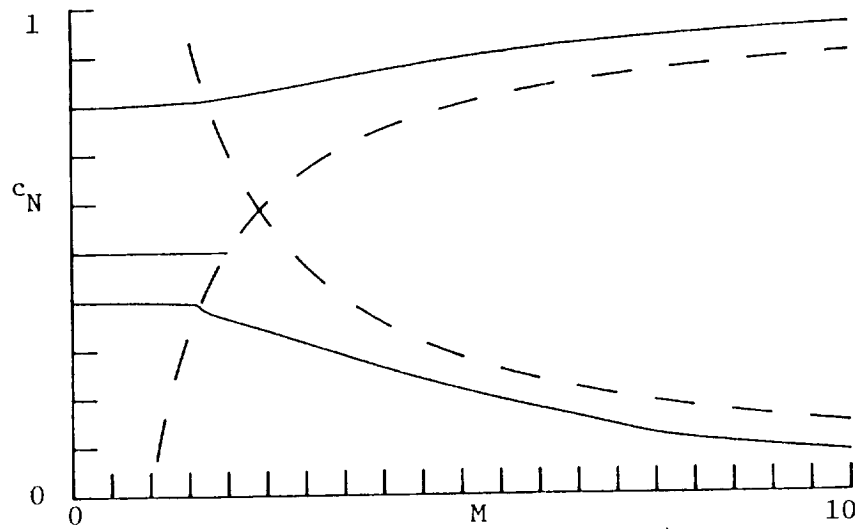


Figure 8. Plot of neutral phase speeds versus Mach number for $\beta = 2$.

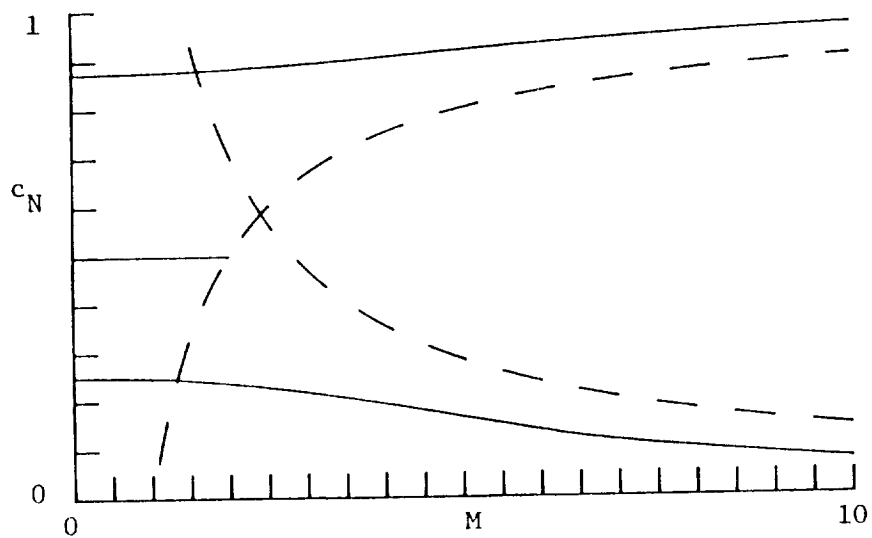


Figure 9. Plot of neutral phase speeds versus Mach number for $\beta = 5$.

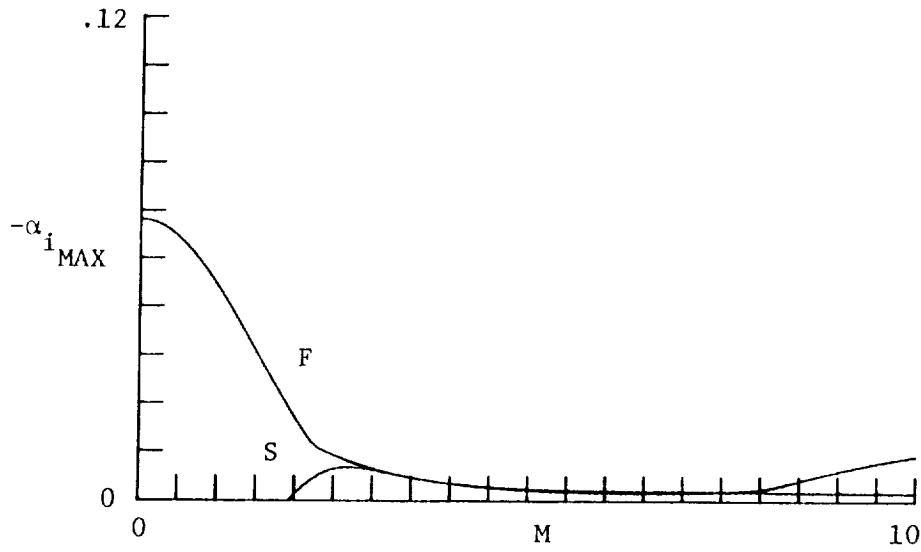


Figure 10. Plot of maximum growth rate versus Mach number for $\beta = 1$.

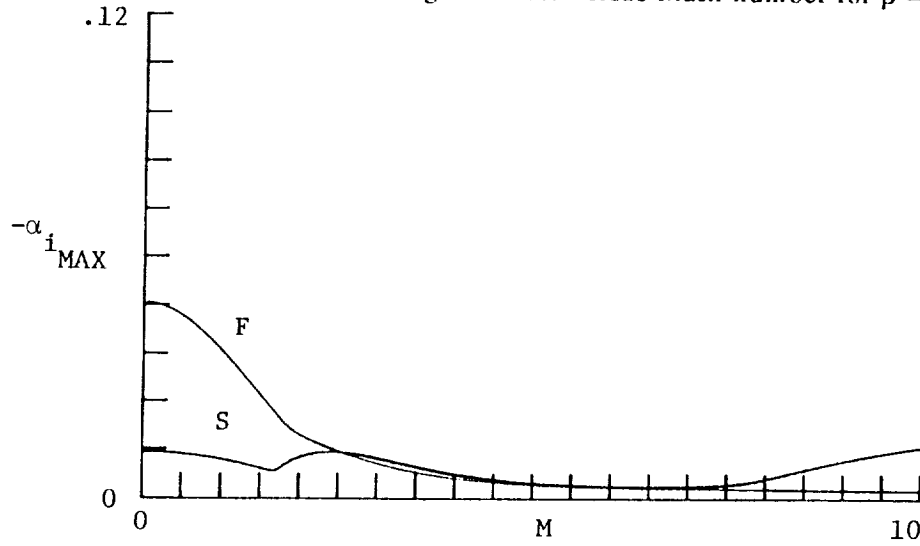


Figure 11. Plot of maximum growth rate versus Mach number for $\beta = 2$.

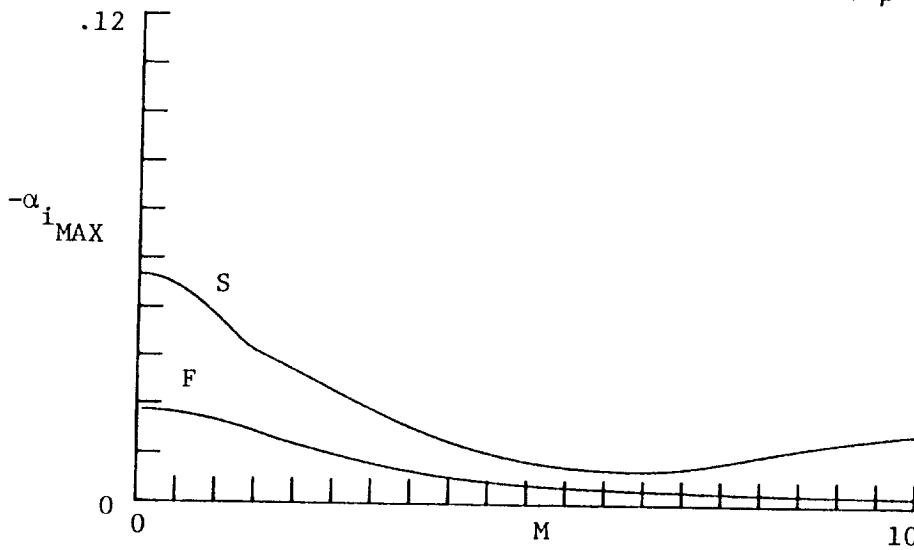


Figure 12. Plot of maximum growth rate versus Mach number for $\beta = 5$.



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